Some Labeling of North Star and Lotus Star Graphs

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Abstract

Two new graphs namely the North Star graph and the Lotus Star graph are introduced in this article. It aims to focus on some labeling methods on both of the graphs. Both the graphs are investigated with a few types of labeling like cordial labeling, sum cordial labeling, signed product cordial labeling and E - sum cordial labeling.

Keywords: Cordial graph, E – sum cordial graph, Lotus Star graph, North Star graph, signed product cordial graph, sum cordial graph.

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1. INTRODUCTION

We begin with simple, finite, undirected graph $G = (V(G), E(G))$, where $V(G)$ and $E(G)$ denotes the vertex set and the edge set respectively. For all other terminology we follow Gross [11]. We provide some useful definitions for the present work.

1.1 Definition

Definition 1.1: The *graph labeling* is an assignment of numbers to the vertices or edges or both subject to certain condition(s). A detailed survey of various graph labeling is explained in Gallian [2].

Definition 1.2: [2] For a graph $G = (V(G), E(G))$, a mapping $f: V(G) \to \{0, 1\}$ is called a *binary vertex labeling* of G and $f(v)$ is called the *label* of the vertex v of G under f. For an edge $e = uv$, the induced edge labeling $f^*: E \to \{0,1\}$ defined as $f^*(uv) = |f(u) - f(v)|$. Let $v_f(0)$ and $v_f(1)$ be the number of vertices of G having labels 0 and 1 respectively under f and let $e_f(0)$ and $e_f(1)$ be the number of edges having labels 0 and 1 respectively under f^*.

Definition 1.3: A binary vertex labeling f of a graph G is called a *cordial labeling* if $|v_f(0) - v_f(1)| \leq 1$ and $|e_f(0) - e_f(1)| \leq 1$. A graph G is said to be *cordial* if it admits cordial labeling.
The notion of cordial labeling was introduced by Cahit [1].

Definition 1.4: A *signed product cordial labeling* of a graph G is a function $f: V(G) \to \{-1, 1\}$ such that each edge uv is assigned the label $f(u)\, f(v)$, the number of vertices with label -1 andthe number of vertices with label 1 differ by atmost 1 and the number of edges with label -1 and thenumber of edges with label 1 differ by atmost 1. A graph which admits signed product cordial labeling is called a *signed product cordial graph*.
The notion of signed product cordial labeling was introduced by Babujee and Loganathan [3].

Definition 1.5: A binary vertex labeling of a graph G with induced edge labeling $f^*: E(G) \to \{0, 1\}$ defined by

$f^*(uv) = (f(u) + f(v))(mod\ 2)$ is called a *sum cordial labeling* if $|v_f(0) - v_f(1)| \leq 1$ and $|e_f(0) - e_f(1)| \leq 1$. A graph G is said to be *sum cordial graph* if it admits sum cordial labeling.
The notion of sum cordial labeling was introduced by Shiama [4].

Definition 1.5: An *E – sum cordial labeling* of a graph G is an edge labeling $f^*: E(G) \rightarrow \{0, 1\}$ with an induced vertex labeling defined by $f(u) = \sum\{f(uv) |\ uv \in E(G)\}\ (mod\ 2)$, where uv are all the incident edges on u, such that it satisfies the condition $|v_f(0) - v_f(1)| \leq 1$ and $|e_f(0) - e_f(1)| \leq 1$. A graph G is said to be *E - sum cordial graph* if it admits *E - sum cordial labeling*.

Origami is an ancient Japanese art of folding paper. The word origami comes from two Japanese words: “ori”, which means to fold, and “kami”, which means paper. Usually, origami models are made strictly by folding paper. There is no cutting or gluing involved. Even if origami is mainly an artistic product, it has received a great deal of attention from mathematicians, because of its interesting algebraic and geometrical properties. We present two new graphs inspired from a model of origami namely North Star and Lotus Star. Also we defined [5] a shining alice graph, [6] a holiday star graph, [7] a Kusudama flower graph, [8] a boreale star graph, [9] a christmas star graph and [10] a braided star graph; We tested various labeling methods on it.

1.2 Construction of North Star graph (NS_n)***:***

Let $v_1, v_2, \ldots, v_{6n-1}, v_{6n}$ be consecutive $6k$ vertices of cycle graph C_{6n}. We form a star graph $K_{1,n}$ using an additional vertex v_0, call it an apex vertex and end vertices as $v_1, v_7, \ldots, v_{6n-11}, v_{6n-5}$ from the cycle C_{6n}. Introducing new vertices u_i such that u_i is adjacent to v_{6i-4}, v_{6i-2} and v_{6i}, for each $i \in [k]$. The resulting graph is called North Star graph NS_n.

$V(NS_n) = \{v_0\} \cup \{v_i \mid i \in [6k]\} \cup \{u_i \mid i \in [k]\}$ and

$E(NS_n) = \{g_i = v_0v_{6i-5}, f_{l_i} = u_iv_{6i-4}, f_i = u_iv_{6i-2}, f_{r_i} = u_iv_{6i} \mid i \in [k]\} \cup \{e_{i_1} = v_{6i-5}v_{6i-4}, e_{i_2} = v_{6i}v_{6i+1} | i \in [k-1]\} \cup \{e_{n_1} = v_{6n-5}v_{6n-4}, e_{n_2} = v_{6n}v_1\} \cup \{p_{l_i} = v_{6i-4}v_{6i-3}, p_{r_i} = v_{6i}v_{6i-1} | i \in [k]\} \cup \{q_{l_i} = v_{6i-3}v_{6i-2}, q_{r_i} = v_{6i-2}v_{6i-1} | i \in [k]\}$.

Thus, $|V(NS_n)| = 7k + 1$ and $|E(NS_n)| = 10k$.

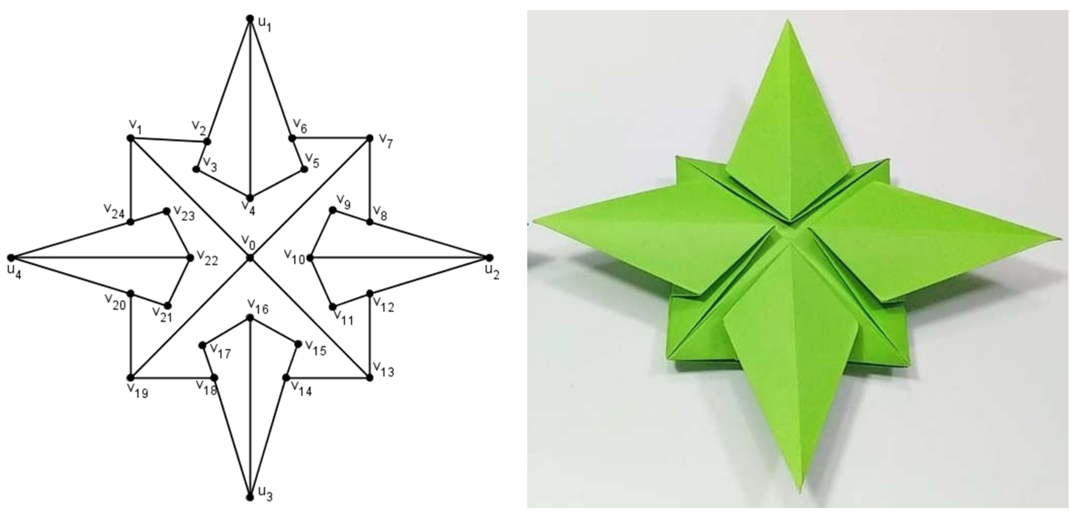

Figure 1: NS_4 **and Origami** NS_4

1.3 Construction of Lotus star graph (LS_n)***:***

Let $v_1, v_2, \ldots, v_{4n-1}, v_{4n}$ be consecutive $4k$ vertices of cycle graph C_{4n}. We form a star graph $K_{1,n}$ using an additional vertex v_0, call it an apex vertex and end vertices as $v_1, v_5, \ldots, v_{4n-3}$ from the cycle C_{4n}. Introducing new vertices u_i such that u_1 is adjacent to $\{v_1, v_2, v_{4n}\}$ and each u_i is adjacent to $\{v_{4i-2}, v_{4(i-1)}, v_{4i-3} | i = 2, 3, \ldots, k\}$. The resulting graph is called lotus star graph LS_n.

$V(LS_n) = \{v_0\} \cup \{v_i \mid i \in [4k]\} \cup \{u_i \mid i \in [k]\}$ and

$E(LS_n) = \{g_i = v_0v_{4i-3},\ f_i = v_{4i-3}u_i |\ i \in [k]\} \cup \{e_{i_1} = v_{4i-3}v_{4i-2}, e_{i_2} = v_{4i-2}v_{4i-1}, e_{i_3} = v_{4i-1}v_{4i}, e_{i_4} = v_{4i}v_{4i+1} |\ i \in [k-1]\} \cup \{e_{n_1} = v_{4n-3}v_{4n-2}, e_{n_2} = v_{4n-2}v_{4n-1}, e_{n_3} = v_{4n-1}v_{4n}, e_{n_4} = v_{4n}v_1\} \cup \{f_{r_i} = u_iv_{4i-2} | i \in [k]\} \cup \{f_{l_{i+1}} = u_{i+1}v_{4i} | i \in [k-1]\} \cup \{f_{l_1} = u_1v_{4n}\}$.

Thus, $|V(LS_n)| = 5k + 1$ and $|E(LS_n)| = 8k$.

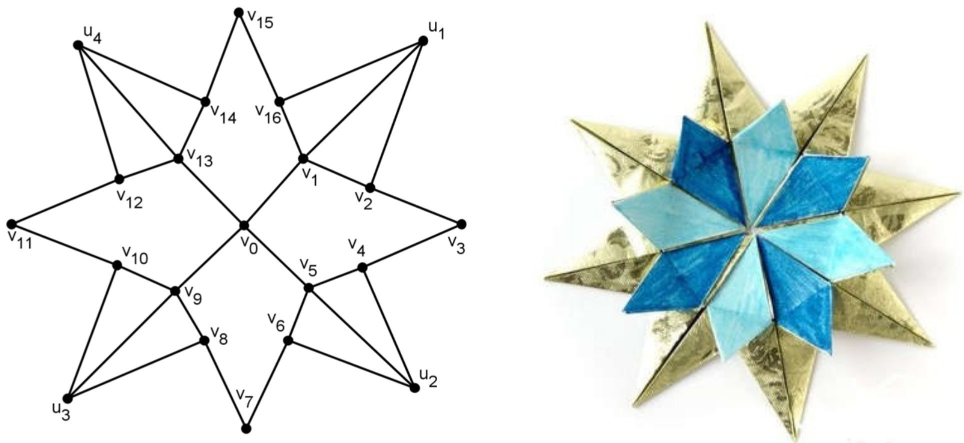

Figure 2: LS_4 **and Origami** LS_8

2. RESULT AND DISCUSSION

Theorem 2.1: NS_n is a signed product cordial graph when k is even and $k \geq 3$.

Proof: Case 1: $k \equiv 0\ (mod\ 4)$

Define an injective function $f: V(NS_n) \to \{-1, 1\}$ as follows:

$$f(x) = \begin{cases} -1, & if\ x = v_0; \\ 1, & if\ x = u_i, i = 2k-1, k \in [\frac{k}{2}]; \\ -1, & if\ x = u_i, i = 2k, k \in [\frac{k}{2}]; \\ 1, & if\ x = v_i, i = 6k-4, k \in [k]; \\ -1, & if\ x = v_i, i = 6k, k \in [k]; \\ 1, & if\ x = v_i, i = 12k-9, k \in [\frac{k}{2}]; \\ -1, & if\ x = v_i, i = 12k-3, k \in [\frac{k}{2}]; \\ 1, & if\ x = v_i, i = 12k-1, k \in [\frac{k}{2}]; \\ -1, & if\ x = v_i, i = 12k-7, k \in [\frac{k}{2}]; \\ -1, & if\ x = v_i, i = 12k-11, k \in [\frac{k}{2}]; \\ 1, & if\ x = v_i, i = 12k-5, k \in [\frac{k}{2}]; \\ 1, & if\ x = v_i, i = 24k-20, k \in [\frac{k}{4}]; \\ 1, & if\ x = v_i, i = 24k-14, k \in [\frac{k}{4}]; \\ -1, & if\ x = v_i, i = 24k-8, k \in [\frac{k}{4}]; \\ -1, & if\ x = v_i, i = 24k-2, k \in [\frac{k}{4}]. \end{cases}$$

Thus, by above labeling we obtain $v_f(1) = \frac{7n+2}{2}$ and $v_f(-1) = \frac{7n}{2}$. Also, $e_f(1) = e_f(-1) = 5k$.

Case 2: $k \equiv 2\ (mod\ 4)$

Define an injective function $f: V(NS_n) \rightarrow \{-1, 1\}$ as follows:

$$f(x) = \begin{cases} -1, & if\ x = v_0; \\ 1, & if\ x = u_i, i = 2k-1, k \in [\frac{k}{2}]; \\ -1, & if\ x = u_i, i = 2k, k \in [\frac{k}{2}]; \\ 1, & if\ x = v_i, i = 6k-4, k \in [k]; \\ -1, & if\ x = v_i, i = 6k, k \in [k]; \\ 1, & if\ x = v_i, i = 12k-9, k \in [\frac{k}{2}]; \\ -1, & if\ x = v_i, i = 12k-3, k \in [\frac{k}{2}]; \\ 1, & if\ x = v_i, i = 12k-1, k \in [\frac{k}{2}]; \\ -1, & if\ x = v_i, i = 12k-11, k \in [\frac{k}{2}]; \\ 1, & if\ x = v_i, i = 12k-5, k \in [\frac{k}{2}]; \\ -1, & if\ x = v_i, i = 12k-7, k \in [\frac{k}{2}]; \\ 1, & if\ x = v_i, i = 24k-20, k \in [\frac{k+2}{4}]; \\ 1, & if\ x = v_i, i = 24k-14, k \in [\frac{k+2}{4}]; \\ -1, & if\ x = v_i, i = 24k-8, k \in [\frac{k-2}{4}]; \\ -1, & if\ x = v_i, i = 24k-2, k \in [\frac{k-2}{4}]. \end{cases}$$

Thus, by above labeling we obtain $v_f(1) = \frac{7n+2}{2}$ and $v_f(-1) = \frac{7n}{2}$. Also, $e_f(1) = e_f(-1) = 5k$. Hence, f satisfies the condition $|v_f(-1) - v_f(1)| \leq 1$ and $|e_f(-1) - e_f(1)| \leq 1$. So, f admits signed product cordial labeling for NS_n. It is a signed product cordial graph.

Theorem 2.2: LS_n is a signed product cordial graph for $k \geq 3$.

Proof: Case 1: k is even

Define an injective function $f: V(LS_n) \rightarrow \{-1, 1\}$ as follows:

$$f(x) = \begin{cases} -1, & if\ x = v_0; \\ -1, & if\ x = v_{4i-3}, i \in [k]; \\ 1, & if\ x = u_i, i \in [k]; \\ 1, & if\ x = v_{4i-2}, i \in [k]; \\ -1, & if\ x = v_{4i}, i \in [k]; \\ 1, & if\ x = v_{8i-5}, i \in [\frac{k}{2}]; \\ -1, & if\ x = v_{8i-1}, i \in [\frac{k}{2}]. \end{cases}$$

Thus, by above labeling we obtain $v_f(-1) = \lfloor\frac{5n+1}{2}\rfloor$ and $v_f(1) = \lceil\frac{5n+1}{2}\rceil$. Also, $e_f(1) = e_f(-1) = 4k$.

Case 2: k is odd.

Define an injective function $f: V(LS_n) \rightarrow \{-1, 1\}$ as follows:

$$f(x) = \begin{cases} -1, & if\ x = v_0; \\ -1, & if\ x = v_{4i-3}, i \in [k]; \\ 1, & if\ x = u_i, i \in [k]; \\ 1, & if\ x = v_{4i-2}, i \in [k]; \\ -1, & if\ x = v_{4i}, i \in [k]; \\ 1, & if\ x = v_{8i-5}, i \in [\frac{k+1}{2}]; \\ -1, & if\ x = v_{8i-1}, i \in [\frac{k-1}{2}]. \end{cases}$$

Thus, by above labeling we obtain $v_f(-1) = v_f(1) = \frac{5n+1}{2}$. Also, $e_f(1) = e_f(-1) = 4k$. Hence, f satisfies the condition $|v_f(-1) - v_f(1)| \leq 1$ and $|e_f(-1) - e_f(1)| \leq 1$. So, f admits signed product cordial labeling for LS_n. It is a signed product cordial graph.

Theorem 2.3: NS_n is a cordial graph for $k \geq 3$.
Proof: Define $f: V(NS_n) \to \{0, 1\}$ as follows:

$$f(x) = \begin{cases} 0, & if\ x = v_0; \\ 1, & if\ x \in \{v_{6i-5}, v_{6i-4}, u_i \mid i \in [k]\}; \\ 0, & if\ x \in \{v_{6i-2}, v_{6i-1}, v_{6i} \mid i \in [k]\}; \\ \frac{1 + (-1)^{i+1}}{2}, & if\ x \in \{v_{6i-3} \mid i \in [k]\}. \end{cases}$$

Thus, $v_f(1) = \rceil\frac{7n}{2}\rceil$ and $v_f(0) = \llcorner\frac{7n}{2} + 1[$.
The induced edge labeling $f^*: E(NS_n) \to \{0, 1\}$ is $f^*(uv) = |f(u) - f(v)|$, for every edge $e = uv \in E$.
Thus,

$$f^*(e) = \begin{cases} 0, & if\ e \in \{e_{i_1}, f_{l_i}, q_{r_i}, p_{r_i} \mid i \in [k]\}; \\ 1, & if\ e \in \{g_i, f_i, f_{r_i} \mid i \in [k]\}; \\ 1, & if\ e \in \{e_{i_2} \mid i \in [k-1]\}; \\ 1, & if\ e = e_{n_2}; \\ \frac{1 + (-1)^{i+1}}{2}, & if\ e \in \{q_{l_i} \mid i \in [k]\}; \\ \frac{1 + (-1)^{i}}{2}, & if\ e \in \{p_{l_i} \mid i \in [k]\}. \end{cases}$$

Thus, $e_f(1) = e_f(0) = 5k$. Hence, f satisfies the condition $|v_f(0) - v_f(1)| \leq 1$ and $|e_f(0) - e_f(1)| \leq 1$. So, f admits cordial labeling for NS_n. It is a cordial graph.

Theorem 2.4: LS_n is a cordial graph for $k \geq 3$.
Proof: Define $f: V(LS_n) \to \{0, 1\}$ as follows:

$$f(x) = \begin{cases} 1, & if\ x = v_0; \\ 1, & if\ x \in \{v_{4i-3}, v_{4i} \mid i \in [k]\}; \\ 0, & if\ x \in \{u_i, v_{4i-2} \mid i \in [k]\}; \\ \frac{1 + (-1)^{i}}{2}, & if\ x \in \{v_{4i-1} \mid i \in [k]\}. \end{cases}$$

Thus, $v_f(0) = \rceil\frac{5n}{2}\rceil$ and $v_f(1) = \llcorner\frac{5n}{2} + 1[$.
The induced edge labeling $f^*: E(LS_n) \to \{0, 1\}$ is $f^*(uv) = |f(u) - f(v)|$, for every edge $e = uv \in E$.
Thus,

$$f^*(e) = \begin{cases} 0, & if\ e \in \{g_i, f_{r_i} \mid i \in [k]\}; \\ 0, & if\ e \in \{e_{i_4} \mid i \in [k]\}; \\ 1, & if\ e \in \{f_i, f_{l_i}, e_{i_1} \mid i \in [k]\}; \\ \frac{1+(-1)^i}{2}, & if\ e \in \{e_{i_2} \mid i \in [k]\}; \\ \frac{1+(-1)^{i+1}}{2}, & if\ e \in \{e_{i_3} \in [k]\}. \end{cases}$$

Thus, $e_f(1) = e_f(0) = 4k$. Hence, f satisfies the condition $|v_f(0) - v_f(1)| \leq 1$ and $|e_f(0) - e_f(1)| \leq 1$. So, f admits cordial labeling for LS_n. It is a cordial graph.

Theorem 2.5: NS_n is an E - sum cordial graph for $k \equiv 0, 1\ (mod\ 4)$ and $k \geq 3$.
Proof: Case 1: $k \equiv 1\ (mod\ 4)$.
Define $f^*: E(NS_n) \to \{0, 1\}$, for every edge $uv \in E$ as follows:

$$f^*(e) = \begin{cases} 0, & if\ e \in \{f_i, f_{l_i}, p_{l_i}, q_{l_i} \mid i \in [k]\}; \\ 1, & if\ e \in \{g_i, f_{r_i}, p_{r_i}, q_{r_i} \mid i \in [k]\}; \\ 0, & if\ e \in \{e_{i_1} \mid i \in [\frac{k+1}{2}]\}; \\ 1, & if\ e \in \{e_{i_2} \mid i \in [\frac{k+1}{2}]\}; \\ 0, & if\ e \in \{e_{i_1} \mid i = \frac{k+5}{2}, \frac{k+9}{2}, \dots, k\}; \\ 1, & if\ e \in \{e_{i_2} \mid i = \frac{k+5}{2}, \frac{k+9}{2}, \dots, k\}; \\ 1, & if\ e \in \{e_{i_1} \mid i = \frac{k+3}{2}, \frac{k+7}{2}, \dots, k-1\}; \\ 0, & if\ e \in \{e_{i_2} \mid i = \frac{k+3}{2}, \frac{k+7}{2}, \dots, k-1\}. \end{cases}$$

Thus, $e_f(1) = e_f(0) = 5k$. The induced vertex labeling $f: V(NS_n) \to \{0, 1\}$ is $f^*(u) = \sum f(uv)\ (mod\ 2)$, for every vertex $v \in V$. Thus, $v_f(0) = v_f(1) = \frac{7n+1}{2}$.

Case 2: $k \equiv 0\ (mod\ 4)$.
Define $f^*: E(NS_n) \to \{0, 1\}$, for every edge $uv \in E$ as follows:

$$f^*(e) = \begin{cases} 0, & if\ e \in \{f_i, f_{l_i}, p_{l_i}, q_{l_i} \mid i \in [k]\}; \\ 1, & if\ e \in \{g_i, f_{r_i}, p_{r_i}, q_{r_i} \mid i \in [k]\}; \\ 0, & if\ e \in \{e_{i_1} \mid i \in [\frac{k}{2}]\}; \\ 1, & if\ e \in \{e_{i_2} \mid i \in [\frac{k}{2}]\}; \\ 0, & if\ e \in \{e_{i1} \mid i = \frac{k+4}{2}, \frac{k+8}{2}, \dots, k\}; \\ 1, & if\ e \in \{e_{i_2} \mid i = \frac{k+4}{2}, \frac{k+8}{2}, \dots, k\}; \\ 1, & if\ e \in \{e_{i_1} \mid i = \frac{k+2}{2}, \frac{k+6}{2}, \dots, k-1\}; \\ 0, & if\ e \in \{e_{i_2} \mid i = \frac{k+2}{2}, \frac{k+6}{2}, \dots, k-1\}. \end{cases}$$

Thus, $e_f(1) = e_f(0) = 5k$. The induced vertex labeling $f: V(NS_n) \rightarrow \{0, 1\}$ is $f^*(u) = \sum f(uv)\ (mod\ 2)$, for every vertex $v \in V$. Thus, $v_f(0) = \lfloor \frac{7n+1}{2} \rfloor$ and $v_f(1) = \lceil \frac{7n+1}{2} \rceil$.

Hence, f satisfies the condition $|v_f(0) - v_f(1)| \leq 1$ and $|e_f(0) - e_f(1)| \leq 1$ for both the cases. So, G admits cordial labeling for NS_n. It is an E - sum cordial graph.

Theorem 2.6: LS_n is an E - sum cordial graph for $k \equiv 0, 2, 3\ (mod\ 4)$ and $k \geq 3$.
Proof: Case 1: k is even.
Define $f^*: E(LS_n) \rightarrow \{0, 1\}$, for every edge $uv \in E$ as follows:

$$f^*(e) = \begin{cases} 0, & if\ e \in \{e_{i_1}, e_{i_4}, f_{l_i} | i \in [k]\}; \\ 1, & if\ e \in \{g_i, f_i, f_{r_i} \mid i \in [k]\}; \\ 0, & if\ e \in \{e_{i_2} \mid i \in [k - \lfloor \frac{k}{4} \rfloor]\}; \\ 1, & if\ e \in \{e_{i_3} \mid i \in [k - \lfloor \frac{k}{4} \rfloor]\}; \\ 1, & if\ e \in \{e_{i_2} | i = k + 1 - \lfloor \frac{k}{4} \rfloor, k + 2 - \lfloor \frac{k}{4} \rfloor, \dots, k\}; \\ 0, & if\ e \in \{e_{i_3} | i = k + 1 - \lfloor \frac{k}{4} \rfloor, k + 2 - \lfloor \frac{k}{4} \rfloor, \dots, k\}. \end{cases}$$

Thus, $e_f(1) = e_f(0) = 4k$. The induced vertex labeling $f: V(LS_n) \rightarrow \{0, 1\}$ is $f(u) = \sum f(uv)\ (mod\ 2)$, for every vertex $v \in V$. Thus, $v_f(0) = \frac{5n+2}{2}$ and $v_f(1) = \frac{5n}{2}$ when $k \equiv 0\ (mod\ 4)$ and $v_f(0) = \frac{5n}{2}$ and $v_f(1) = \frac{5n+2}{2}$ when $k \equiv 2\ (mod\ 4)$.
Case 2: $k \equiv 3\ (mod\ 4)$.
Define $f^*: E(LS_n) \rightarrow \{0, 1\}$, for every edge $uv \in E$ as follows:

$$f^*(e) = \begin{cases} 0, & if\ e \in \{e_{i_1}, e_{i_4}, f_{l_i} | i \in [k]\}; \\ 1, & if\ e \in \{g_i, f_i, f_{r_i} \mid i \in [k]\}; \\ 0, & if\ e \in \{e_{i_2} | i \in [\frac{3k-1}{4}]\}; \\ 1, & if\ e \in \{e_{i_3} \mid i \in [\frac{3k-1}{4}]\}; \\ 1, & if\ e \in \{e_{i_2} | i = \frac{3k+3}{4}, \frac{3k+7}{4}, \dots, k\}; \\ 0, & if\ e \in \{e_{i_3} \mid i = \frac{3k+3}{4}, \frac{3k+7}{4}, \dots, k\}. \end{cases}$$

Thus, $e_f(1) = e_f(0) = 4k$. The induced vertex labeling $f: V(LS_n) \rightarrow \{0, 1\}$ is $f(u) = \sum f(uv)\ (mod\ 2)$, for every vertex $v \in V$. Thus, $v_f(0) = \frac{5n+1}{2}$ and $v_f(1) = \frac{5n+1}{2}$.
Hence, f satisfies the condition $|v_f(0) - v_f(1)| \leq 1$ and $|e_f(0) - e_f(1)| \leq 1$ for both the cases. So, G admits cordial labeling for LS_n. It is an E – sum cordial graph.

Theorem 2.7: NS_n is a sum cordial graph for $k \geq 3$.
Proof: Case 1: k is even.
Define $f: V(NS_n) \rightarrow \{0, 1\}$ as follows:

$$f(x)=\begin{cases}1, & if\ x=v_0;\\ 1, & if\ x\in\{v_{6i-5},v_{12i-7}\ |i\in[\frac{k}{2}]\};\\ 0, & if\ x\in\{v_{6i-5}\ |i=\frac{k+2}{2},\frac{k+4}{2},\dots,k\};\\ 1, & if\ x\in\{v_{6i-4},v_{6i-3}\ |i\in[k]\};\\ 0, & if\ x\in\{v_{6i}\ |i\in[k]\};\\ 0, & if\ x\in\{v_{12i-1}\ |i\in[\frac{k}{2}]\};\\ 0, & if\ x\in\{v_{6i-2}\ |i\in[k]\};\\ 0, & if\ x\in\{u_{2i-1}\ |i\in[\frac{k}{2}]\};\\ 1, & if\ x\in\{u_{2i}\ |i\in[\frac{k}{2}]\}.\end{cases}$$

Thus, $v_f(0)=\lceil\frac{7n}{2}\rceil$ and $v_f(1)=\lfloor\frac{7n+2}{2}\rfloor$. Thus, obtained induced edge labeling is $e_f(1)=e_f(0)=5k$.

Case 2: k is odd.

Define $f:V(NS_n)\to\{0,1\}$ as follows:

$$f(x)=\begin{cases}0, & if\ x=v_0;\\ 1, & if\ x\in\{v_{6i-5}|i\in[\lceil\frac{k}{2}\rceil]\};\\ 0, & if\ x\in\{v_{6i-5}\ |i=\lceil\frac{k}{2}\rceil+1,\lceil\frac{k}{2}\rceil+2,\dots,k\};\\ 1, & if\ x\in\{v_{6i-4},v_{6i-3}\ |i\in[k]\};\\ 0, & if\ x\in\{v_{6i},v_{6i-2}\ |i\in[k]\};\\ 1, & if\ x\in\{v_{12i-1}\ |i\in[\lfloor\frac{k}{2}\rfloor]\};\\ 0, & if\ x\in\{v_{12i-7}\ |i\in[\lceil\frac{k}{2}\rceil]\};\\ 1, & if\ x\in\{u_{2i-1}\ |i\in[\lceil\frac{k}{2}\rceil]\};\\ 0, & if\ x\in\{u_{2i}\ |i\in[\lfloor\frac{k}{2}\rfloor]\}.\end{cases}$$

Thus, $v_f(0)=v_f(1)=\frac{7n}{2}$. Thus, obtained induced edge labeling is $e_f(1)=e_f(0)=5k$.

Hence, f satisfies the condition $|v_f(0)-v_f(1)|\le 1$ and $|e_f(0)-e_f(1)|\le 1$. So, f admits sum cordial labeling for NS_n. It is a sum cordial graph.

Theorem 2.8: LS_n is a sum cordial graph for $k\ge 3$.

Proof: Case 1: k is even.

Define $f:V(LS_n)\to\{0,1\}$ as follows:

$$f(x)=\begin{cases}1, & if\ x=v_0;\\ 1, & if\ x\in\{v_{4i-3},v_{4i-2}\ |i\in[k]\};\\ 0, & if\ x\in\{u_i,v_{4i}\ |i\in[k]\};\\ 0, & if\ x\in\{v_{4i-1}|i\in[\frac{k}{2}]\};\\ 1, & if\ x\in\{v_{4i-1}\ |i=\frac{k+2}{2},\frac{k+4}{2},\dots,k\}.\end{cases}$$

Thus, $v_f(1)=\lceil\frac{5n+1}{2}\rceil$ and $v_f(0)=\lfloor\frac{5n+1}{2}\rfloor$. Thus, obtained induced edge labeling is $e_f(1)=e_f(0)=4k$.

Case 2: k is odd.

Define $f:V(LS_n)\to\{0,1\}$ as follows:

$$f(x) = \begin{cases} 1, & if\ x = v_0; \\ 1, & if\ x \in \{v_{4i-3}, v_{4i-2}\ |i \in [k]\}; \\ 0, & if\ x \in \{u_i, v_{4i}\ |i \in [k]\}; \\ 1, & if\ x \in \{v_{4i-1} | i \in [\frac{k-1}{2}]\}; \\ 0, & if\ x \in \{v_{4i-1}\ | i = \frac{k+1}{2}, \frac{k+3}{2}, \dots, k\}. \end{cases}$$

Thus, $v_f(0) = v_f(1) = \frac{5n+1}{2}$. Thus, obtained induced edge labeling is $e_f(1) = e_f(0) = 4k$.

Hence, f satisfies the condition $|v_f(0) - v_f(1)| \leq 1$ and $|e_f(0) - e_f(1)| \leq 1$. So, f admits sum cordial labeling for LS_n. It is a sum cordial graph.

3. CONCLUSION

Using origami, new graphs like North star and Lotus star are introduced. Various graph labeling is applied on both of the graphs. North star and Lotus star graph admits labeling like cordial, sum cordial, E – sum cordial and signed product cordial labeling. Different cases where these graphs do not admit particular labeling is an open problem for the researchers.

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