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Analysis of the dynamics of the resonant planetary system HD 45364

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Abstract The aim of this work is to analyse the orbital dynamics and stability of exoplanetary system in which two planets are in nearly 3:2 MMR. We have taken HD 45364 system for our study in which two planets are most likely in a 3:2 MMR. We have plotted two resonant angles and the relative apsidal longitudes and it is observed that they are librating around a constant value which confirms the existence of nearly 3:2 MMR between HD 45364b and HD 45364c. The perturbative solution is obtained for the time variation of the semi-major axes. The short and long-term variations of semimajor axes are studied. For the validation of our analytical results we have compared the analytical solutions with the numerical solutions. The effect of planetary perturbations on eccentricity is studied with the help of secular resonance dynamics theory. The short and long-term variations of eccentricities of HD 45364b and HD 45364c are shown graphically. Moreover, using recently developed stability criteria, we have studied the dynamical stability of HD 45364 system.

Keywords Three-body problem · MMR · HD 45364 system · Dynamical Stability

1 Introduction

Nowadays the study of exoplanetary systems are hot topic for the researchers who are working in the field of astronomy, astrophysics and celestial mechanics etc. Scientists are trying to find the possibility of existence of life in the planetary systems other than our own solar system. Few planetary systems are discovered containing at least one earth like planet where life can exist. For example Kepler-62

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(Borucki et al, 2013; Mia and Kushvah, 2016a), Kepler-186 (Bolmont et al, 2014), Kepler-69 (Barclay et al, 2013) and HD 40307 (Tuomi et al, 2013) systems consist at least one earth like planet in the habitable zone. Among about 5000 newly discovered exoplanetary systems only few systems found in mean-motion resonances (MMRs). For example HD73526, HD82943 systems are in 2:1 MMR, two planets of HD 200964 system are nearly 4:3 MMR, two planets of Kepler-11 are lie in 5:4 MMR. Higher order MMRs are also exist in exoplanetary systems for example, HD 60532 (3:1 MMR), HD 108874 (4:1 MMR) and HD 102272 (4:1 MMR) (Libert and Tsiganis, 2009). Moreover, in our solar system Saturn–Uranus and Uranus–Neptune systems are in 3:1 and 2:1 near MMRs respectively (Petrovich et al, 2013). There are several scientists who have studied the orbital evolution of resonant planetary systems (Crida et al, 2008; Petrovich et al, 2013; Barnes et al, 2015; Mia and Kushvah, 2016a; Rosenthal et al, 2019).

In HD 45364 planetary system, the host star HD 45364 is near about 107 light years away from Earth. There are two confirmed exoplanets in this sytem namely HD 45364b and HD 45364c respectively orbiting around the star HD 45364. Rein et al (2010) have shown that the two planets in the exoplanetary system HD 45364 are most likely in a 3:2 MMR. Dynamically, HD 45364 system is very much interesting and attractive. The existence of two or three or more interacting planets in a planetary system enhances our possible ability to constrain and understand the processes of planetary formation and evolution excitingly. To determine the system structure in terms of orbital content and for constraining the system evolution history, the dynamical analysis of this type of systems is very useful (Correia et al, 2009).

It was Rodr´ıguez and Gallardo (2005) who have analyzed the probable dynamical mechanisms that govern the motion of the HD 12661 exoplanetary system. Using an analytical approach and the expansion of the disturbing function, they have solved the equation of motion in a Hamiltonian formulation. They have also discussed the occurrence of mean-motion resonances (MMRs) in the system and analyzed the possible contribution from these resonant terms for the total motion.

For better understanding of habitability of any planetary system it is first required to study the stability and dynamics of those planetary systems. There are many researchers who have studied the stability of solar system and exoplanetary systems. Many researchers(Abouelmagd et al, 2013; Pal and Kushvah, 2015; Singh et al, 2016; Mia and Kushvah, 2016b; Singh et al, 2018; Yadav et al, 2021; Abouelmagd et al, 2021; Gyegwe et al, 2022) aslo studied the stability of an infinitesimal particle like asteroid, satellite, dust grain etc., in the solar and exoplanetary systems. For example Petrovich (2015) studied the stability and fates of hierarchical tow planet systems with arbitrary eccentricities and mutual inclinations. They have proposed a new stability criteria and using numerical integrations they have shown that their stability criteria is better that the existence criteria. Later Mia and Kushvah (2016a) and Handayani and Dermawan (2016) successfully applied this new criteria to some resonant exoplanetary systems.

The purpose of this work to analyze the evolution of orbital elements and dynamical stability of HD45364 system in which two planets are nearly in 3:2 MMR. Using the expansion of the disturbing function given by Mia and Kushvah (2016a), we find the perturbation equations of orbital elements for 3:2 MMR and then we solved the equation of motion analytically. Moreover, we have studied the dynamical stability of the system using recently developed stability criteria given by Petrovich (2015).

This paper is structured as follows: In Section 2, we have provided the perturbation equations of orbital elements for 3:2 MMR. We have discussed the 3:2 MMR and the evolution of resonant angles of HD 45364 system in Section 3. The effect of planetary perturbations on semimajor axes are analysed in Section 4. Section 5 devoted to the secular resonance dynamics and effect of planetary perturbations on eccentricity. Furthermore, we have discussed the dynamical stability of HD 45364 system in Section 6. Finally, conclusions are given in Section 7.

2 Perturbation Equations of Orbital Elements

We consider the three-body problem as the model to study the dynamics of HD 45364 planetary system. We assume that two planets having masses M_1 and M_2 orbiting a star of mass M_s. Let us denote the mean motion of the j-th planet by $n_j (j = 1, 2)$ and semimajor axes ratio of planet 1 and planet 2 by β where $\beta < 1$. Let a_j, e_j, ω_j and λ_j are the semimajor axis, eccentricity, longitude of the pericentre and mean longitude for j-th planet$(j = 1, 2)$. Then the perturbation equations for the time variation of the orbital elements for 3:2 MMR are given by Mia and Kushvah (2016a) as follows:

The equation for time variation of the semi-major axes:

$$
\begin{aligned}
\frac{\dot{a}_1}{a_1} = 2\frac{M_2}{M_s} n_1 \beta \, [\partial_\psi Q(\psi, \beta) + \beta \sin \psi \\
-2\left(3 + \frac{\beta}{2} D\right) b^{(3)}_{\frac{1}{2}}(\beta) e_1 \\
\times \sin(3\lambda_2 - 2\lambda_1 - \omega_1) \\
+2\left(\frac{5}{2} + \frac{\beta}{2} D\right) b^{(2)}_{\frac{1}{2}}(\beta) e_2 \\
\times \sin(3\lambda_2 - 2\lambda_1 - \omega_2)],
\end{aligned} \tag{1}
$$

$$
\begin{aligned}
\frac{\dot{a}_2}{a_2} = -2\frac{M_1}{M_s} n_2 \, [\partial_\psi Q(\psi, \beta) + \beta \sin \psi \\
-3\left(3 + \frac{\beta}{2} D\right) b^{(3)}_{\frac{1}{2}}(\beta) e_1 \\
\times \sin(3\lambda_2 - 2\lambda_1 - \omega_1) \\
+3\left(\frac{5}{2} + \frac{\beta}{2} D\right) b^{(2)}_{\frac{1}{2}}(\beta) e_2 \\
\times \sin(3\lambda_2 - 2\lambda_1 - \omega_2)].
\end{aligned} \tag{2}
$$

The equation for time variation of the eccentricities:

$$
\begin{aligned}
\dot{e}_1 = \frac{M_2}{M_s} n_1 \beta \left[\frac{1}{4} \beta b^{(2)}_{\frac{3}{2}}(\beta) e_2 \sin(\omega_1 - \omega_2)\right. \\
-\left(3 + \frac{\beta}{2} D\right) b^{(3)}_{\frac{1}{2}}(\beta) \\
\left.\times \sin(3\lambda_2 - 2\lambda_1 - \omega_1)\right],
\end{aligned} \tag{3}
$$

$$\dot{e}_2 = -\frac{M_1}{M_s} n_2 \left[\frac{1}{4}\beta b^{(2)}_{\frac{3}{2}}(\beta) e_1 \sin(\omega_1 - \omega_2) - \left(\frac{5}{2} + \frac{\beta}{2} D \right) b^{(2)}_{\frac{1}{2}}(\beta) \times \sin(3\lambda_2 - 2\lambda_1 - \omega_2) \right]. \quad (4)$$

The equation for time variation of the periastrons:

$$\dot{\omega}_1 = \frac{M_2}{M_s} n_1 \beta \left[\frac{1}{4}\beta b^{(1)}_{\frac{3}{2}}(\beta) - \frac{1}{4}\beta b^{(2)}_{\frac{3}{2}}(\beta) \frac{e_2}{e_1} \cos(\omega_1 - \omega_2) - \frac{1}{e_1} \left(3 + \frac{\beta}{2} D \right) b^{(3)}_{\frac{1}{2}}(\beta) \times \cos(3\lambda_2 - 2\lambda_1 - \omega_1) \right], \quad (5)$$

$$\dot{\omega}_2 = \frac{M_1}{M_s} n_2 \left[\frac{1}{4}\beta b^{(1)}_{\frac{3}{2}}(\beta) - \frac{1}{4}\beta b^{(2)}_{\frac{3}{2}}(\beta) \frac{e_1}{e_2} \cos(\omega_1 - \omega_2) + \frac{1}{e_2} \left(\frac{5}{2} + \frac{\beta}{2} D \right) b^{(2)}_{\frac{1}{2}}(\beta) \times \cos(3\lambda_2 - 2\lambda_1 - \omega_2) \right]. \quad (6)$$

Where,

$$\lambda_j = \mathrm{M}_j + \omega_j,\ \psi = \lambda_1 - \lambda_2,\ D = \frac{d}{d\beta} \text{ and } Q(\psi, \beta) = (1 - 2\beta \cos\psi + \beta^2)^{-\frac{1}{2}}. \quad (7)$$

Also the Laplace coefficients $b_s^{(j)}(\beta)$ and Fourier series expansion of $Q(\psi, \beta)$ are defined as

$$\frac{1}{2} b_s^{(j)}(\beta) = \frac{1}{2\pi} \int_0^{2\pi} \frac{\cos jp \, dp}{(1 - 2\beta \cos p + \beta^2)^s}, \quad (8)$$

$$Q(\psi, \beta) = \frac{1}{2} \sum_{j=-\infty}^{\infty} b_s^{(j)}(\beta) \cos j\psi. \quad (9)$$

3 The 3:2 Mean motion-resonance(MMR)

We are now going to discuss on the dynamics of 3:2 resonance of planets HD 45364b and HD 45364c orbiting the star HD 45364. From the Table 1, The periods of HD 45364b and HD 45364c are 226.93 and 342.85 respectively. So, there may exists nearly 3:2 MMR between HD 45364b and HD 45364c. The two terms associated with the first order 3 : 2 arguments are $\vartheta_1 = 3\lambda_2 - 2\lambda_1 - \omega_1$ and $\vartheta_2 = 3\lambda_2 - 2\lambda_1 - \omega_2$, where ω_1, ω_2 are periastron longitudes and λ_1, λ_2 are the mean longitudes of HD 45364b and HD 45364c, respectively. For the resonant systems, the apsidal lock between the orbiting companions is also an another important feature where the relative apsidal longitudes is defined as

$$\Delta\omega = \omega_1 - \omega_2 \quad (10)$$

Now if we find that atleast one of the resonant angles(ϑ_1 or ϑ_2) librates about a constant value, we say that planets HD 45364b and HD 45364c are in 3:2 mean

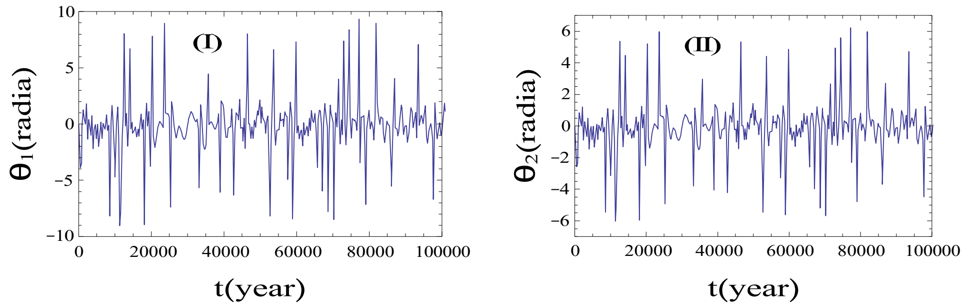

Fig. 1: The evolution of the resonant angles ϑ_1 (I) and ϑ_2 (II) of HD 45364 system.

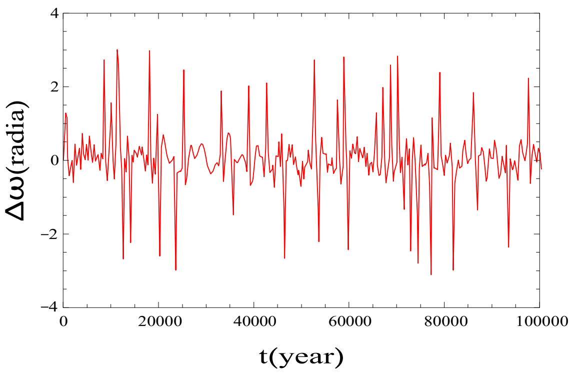

Fig. 2: The evolution of the apsidal angle $\Delta\omega$ of HD 45364 system.

motion-resonance and if $\Delta\omega$ also librates then the system is said to be in apsidal co-rotation. We have plotted the evolution of the resonant angles ϑ_1 and ϑ_2 and apsidal angle $\Delta\omega$ for long time $t \in [0, 100, 000]$. The resonant angles ϑ_1 and ϑ_2 are depicted in Fig. 1 and the apsidal angle $\Delta\omega$ is depicted in Fig. 2. From Fig. 1, it is clear that the resonant angle ϑ_1 is librating about 0 rad with the peak-to-valley amplitude of libration of ±8 rad and the resonant angle ϑ_2 is librating about 0 rad with the peak-to-valley amplitude of libration of ±5 rad. From these results, we can say that the two planets HD 45364b and HD 45364c are in nearly 3:2 MMR. Also from Fig. 2, it is obvious that the apsidal angle $\Delta\omega$ librates around 0 rad with an amplitude of libration of ±3 rad, which indicates that the system is in apsidal co-rotation.

4 The effect of planetary perturbations on semimajor axis

Now to obtain the variations of the semimajor axes, we integrate the Eqs. (1) and (2) and then variations of the semimajor axes are obtained as $a_1(t) = a_{1,0} + \delta a_1(t)$,

Table 1: Orbital parameters of the HD 45364 system: the data are taken from Correia et al (2009)

Parameter	HD 45364	HD 45364b	HD 45364c
Mass	0.82 $M_\odot$	0.1872 M_J	0.6579 M_J
Period(d)		226.93	342.85
a(AU)		0.6813	0.8972
e		0.1684	0.0974
ω(deg)		162.58	7.41
Epoch(JD)		2453500	2453500

$a_2(t) = a_{2,0} + \delta a_2(t)$, where

$$\frac{\delta a_1(t)}{a_{1,0}} = \frac{2M_2\beta}{M_s}\left(\frac{n_1}{n_1-n_2}\left[Q(\psi(t),\beta) - Q(\psi_0,\beta)\right.\right.$$
$$\left.-\beta(\cos\psi(t) - \cos\psi_0)\right] - \frac{2n_1}{3n_2-2n_1}\left(-\left(3+\frac{\beta}{2}D\right)\right.$$
$$\times b^{(3)}_{\frac{1}{2}}(\beta)e_1(\cos\vartheta_1(t) - \cos\vartheta_{1,0}) + \left(\frac{5}{2}+\frac{\beta}{2}D\right)$$
$$\left.\left.\times b^{(2)}_{\frac{1}{2}}(\beta)e_2(\cos\vartheta_2(t) - \cos\vartheta_{2,0})\right)\right), \tag{11}$$

$$\frac{\delta a_2(t)}{a_{2,0}} = -\frac{2M_1}{M_s}\left(\frac{n_2}{n_1-n_2}\left[Q(\psi(t),\beta) - Q(\psi_0,\beta)\right.\right.$$
$$\left.-\beta(\cos\psi(t) - \cos\psi_0)\right] - \frac{3n_2}{3n_2-2n_1}\left(-\left(3+\frac{\beta}{2}D\right)\right.$$
$$\times b^{(3)}_{\frac{1}{2}}(\beta)e_1(\cos\vartheta_1(t) - \cos\vartheta_{1,0}) + \left(\frac{5}{2}+\frac{\beta}{2}D\right)b^{(2)}_{\frac{1}{2}}(\beta)$$
$$\left.\left.\times e_2(\cos\vartheta_2(t) - \cos\vartheta_{2,0})\right]\right), \tag{12}$$

where

$$\vartheta_j(t) = (3n_2 - 2n_1)t + 3(\sigma_2 + \omega_2) - 2(\sigma_1 + \omega_1) - \omega_j, \qquad (j = 1,\ 2)$$
$$\psi(t) = (n_1 - n_2)t + (\sigma_1 + \omega_1) - (\sigma_2 + \omega_2). \tag{13}$$

Using the data given in the Table 1, we have $n_1 = \frac{2\pi}{226.93}$ rad/d and $n_2 = \frac{2\pi}{342.85}$ rad/d. It is clear from Eqs. (11) and (12) that there are two components for the variations in the semimajor axes. The period in the first component is $\frac{2\pi}{|n_1-n_2|} \approx 671.178$ days and corresponding fractional amplitude $\frac{M_J n_1}{M_\odot|n_1-n_2|} \approx 0.00226326$ while in the second component period is $\frac{2\pi}{|3n_2-2n_1|} \approx 15845.8$ days with fractional amplitude $\frac{M_J n_1}{M_\odot|3n_2-2n_1|} \approx 0.0534333$.

The time variation of semimajor axis of HD 45364b is shown in Fig 3. In this figure curve (I) represents the variation of semimajor axis for the time interval $t \in (0, 500)$ and curve (II) represents the variation of semimajor axis for long time $t \in (0, 10000)$. Likewise, the time variation of semimajor axis of HD 45364c is depicted in Fig. 4. In Fig. 4, curve (I) represents the time variation of semimajor axis of HD 45364c for the time interval $t \in (0, 500)$ while curve (II) represents the time variation of semimajor axis of HD 45364c for long time $t \in (0, 10000)$.

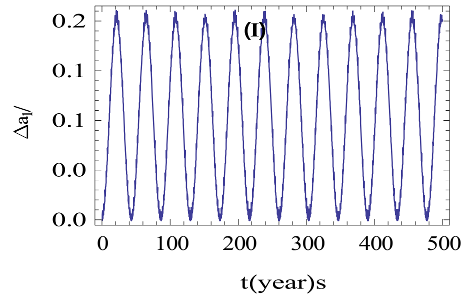

Fig. 3: Perturbative solution for the time variation of the semi-major axes: (I) for $t \in [0, 500]$, (II) for long time $t \in [0, 10000]$ of inner planet HD 45364b.

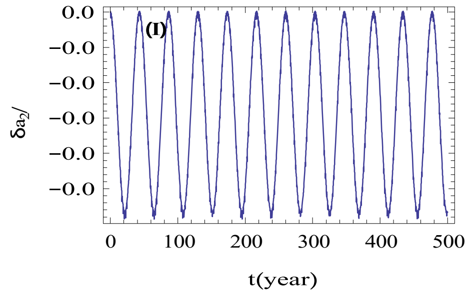

Fig. 4: Perturbative solution for the time variation of the semi-major axes: (I) for $t \in [0, 500]$, (II) for long time $t \in [0, 10000]$ of outer planet HD 45364c.

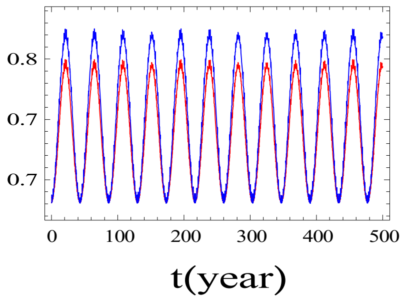

Fig. 5: Comparison between numerical and analytical solution for the time variation of the semi-major axes of HD 45364 system: the blue line indicates the result by analytical method and the red line indicates solution obtained by numerical method.

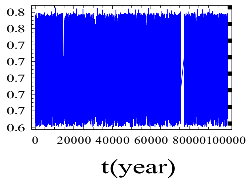

Fig. 6: Comparison between numerical and analytical solution of the semimajor axes of planets of HD 45364 system for long time $t \in [0, 100000]$. The upper panel corresponds to HD 45364b and the lower panel corresponds to HD 45364c. In each panel, curve (I) shows the result by analytical theory, curve (II) shows the numerical solution while curve (III) represents the zoom part of (I) and (II) for the time interval $t \in [99500, 100000]$.

Furthermore to validate the analytical results, we have made a comparison between numerical and analytical solution for the time variation of the semimajor axes which is shown in Fig. 5. In this Fig. 5, the blue line represents the results obtained by analytical theory and the red line represents the solution obtained by numerical method. Also for long time $t \in (0, 10000)$, a comparison between numerical and analytical solution for the time variation of the orbital semimajor axes of HD 45364b and HD 45364c are depicted in Fig. 6. In this Fig. 6, the upper panel is for the inner planet HD 45364b and the lower panel is for the outer planet HD 45364c. In both the panels, curve (I) and curve (II) shows the result by analytical method and numerical methods respectively. In this figure curve (III) represents the variation of semimajor axes for $t \in (99500, 100000)$ which is actually the zoom part of frames (I) and (II) for the time interval $t \in [99500, 100000]$. We have observed that for long time, analytical solution of semimajor axis of the inner planet HD 45364b lies in the interval [0.68, 0.82] and numerical solution lies in the interval [0.68, 0.792] and in the case of HD 45364c analytical solution lies in the interval [0.846, 0.898] and numerical solution lies in the interval [0.854, 0.896].

5 Secular resonance dynamics and effect of planetary perturbations on eccentricity

Now we are going to discuss the long-term variations of the eccentricities of HD 45364b and HD 45364c with the help of secular theory with MMR. For $r + 1 : r$ MMR, the solutions for the eccentricities are given by (Mia and Kushvah, 2016c)

$$p_j(t) = \sum_{i=1}^{2} e_{ji} \sin(g_i t + \beta_i) + F_j \sin((r+1)\lambda_2 - r\lambda_1),$$
$$q_j(t) = \sum_{i=1}^{2} e_{ji} \cos(g_i t + \beta_i) + F_j \cos((r+1)\lambda_2 - r\lambda_1). \quad (14)$$

In our case for 3:2 MMR, the solutions of the eccentricities are obtained from the above two equations as

$$
\begin{aligned}
p_j(t) &= \sum_{i=1}^{2} e_{ji} \sin(g_i t + \beta_i) + F_j \sin(3\lambda_2 - 2\lambda_1),\\
q_j(t) &= \sum_{i=1}^{2} e_{ji} \cos(g_i t + \beta_i) + F_j \cos(3\lambda_2 - 2\lambda_1),
\end{aligned} \tag{15}
$$

where the frequencies g_i ($i = 1, 2$) are the eigenvalues of the following coefficient matrix A

$$
\begin{bmatrix}
\frac{M_2}{4M_s} n_1 \beta^2 b^{(1)}_{\frac{3}{2}}(\beta) & -\frac{M_2}{4M_s} n_1 \beta^2 b^{(2)}_{\frac{3}{2}}(\beta) \\
-\frac{M}{4M_s} n_2 \beta b^{(2)}_{\frac{3}{2}}(\beta) & \frac{M_1}{4M_s} n_2 \beta b^{(1)}_{\frac{3}{2}}(\beta)
\end{bmatrix}
$$

and e_{ji} are the component of the two corresponding eigenvectors. Moreover the vertical component p_j and horizontal component q_j of eccentricity are given by

$$
p_j = e_j \sin \omega_j, \qquad q_j = e_j \cos \omega_j. \tag{16}
$$

The normalization of eigenvectors and phases β_i can be determined by the initial conditions. The amplitude of forcing is given as

$$
\begin{pmatrix} F_1 \\ F_2 \end{pmatrix} = -B^{-1}. \begin{pmatrix} E_1 \\ E_2 \end{pmatrix}, \tag{17}
$$

where $B = [A - \{3n_2 - 2n_1\}I]$ and I denotes a 2×2 identity matrix. Also

$$
\begin{aligned}
E_1 &= -\frac{M_2}{M_s} n_1 \beta \left(3 + \frac{\beta}{2} D\right) b^{(3)}_{\frac{1}{2}}(\beta),\\
E_2 &= \frac{M_1}{M_s} n_2 \left(\frac{5}{2} + \frac{\beta}{2} D\right) b^{(2)}_{\frac{1}{2}}(\beta).
\end{aligned} \tag{18}
$$

With the numerical values of the orbital parameters from the Table 1 and using the theory discussed above, for HD 45364 system, we obtain $g_1 = 2.27956 \times 10^{-2}$rad yr^{-1}and $g_2 = 1.6483 \times 10^{-3}$rad yr^{-1} together with $\beta_1 = 0.479303$ rad, $\beta_2 = -0.857702$ rad, and $F_1 = 7.23968 \times 10^{-2}$, $F_2 = -1.82078 \times 10^{-2}$.

The evolution of the eccentricities of the two planets HD 45364b and HD 45364c are shown in Fig. 7 over a time span of 10 yr. From this Fig. 7, we can observe that the periodicity occurs in the variation of eccentricities for both the planets. It is also found that eccentricities for both the planets reach its maximum value at the same time and conversely when one eccentricity is minimum, then other reaches exactly its minimum value. Moreover, the eccentricity of HD 45364b oscillates between 0.07 and 0.2147 and eccentricity of HD 45364c oscillates between 0.0976 and 0.1363. We have also shown the long-term variation of the eccentricities over a time span of 10000 yrs. in Fig. 8 and we observe the similar types of behaviours in the variation of eccentricities of HD 45364b and HD 45364c for long time also.

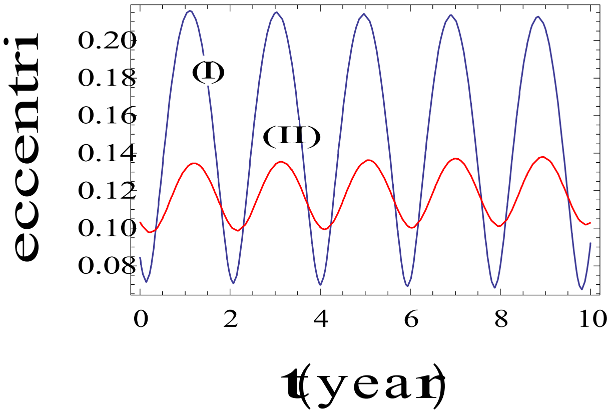

Fig. 7: Planet's eccentricity: curve (I) is eccentricity of HD 45364b , curve (II) is eccentricity of HD 45364c for the time $t \in [0, 10]$.

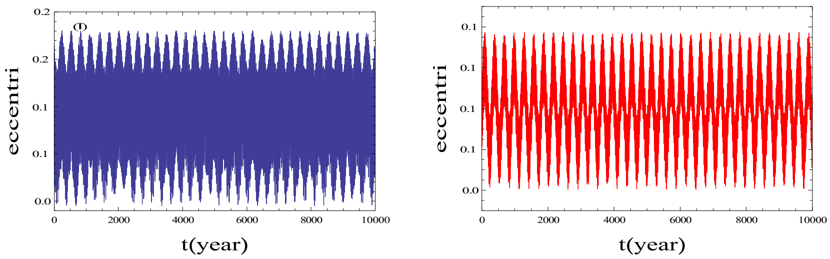

Fig. 8: Planet eccentricity: (I) represents the eccentricity of HD 45364b, (II) represents the eccentricity of HD 45364c for long time $t \in [0, 10000]$.

6 Empirical stability criteria and dynamical stability of HD 45364 system

Now we discuss the dynamical stability of HD 45364 system. It was Petrovich (2015) who have studied an independent review on the stability of two-planet systems and given one latest empirical stability criteria for resonant planetary systems. They have used numerical integration to test other stability criteria that have been proposed by other researchers and shown that their stability criteria perform significantly better. They have given the following new stability criteria for dynamical stability planetary systems

$$rap = \frac{a_2(1-e_2)}{a_1(1+e_1)} > Y = 2.4[\max(\mu_1, \mu_2)]^{\frac{1}{3}} \left(\frac{a_2}{a_1}\right)^{\frac{1}{2}} + 1.15, \qquad (19)$$

This criteria can be applied to planet-star mass ratios $\mu_1, \mu_2 \in [10^{-4}, 10^{-2}]$ where, μ_1, μ_2 are the planet to star mass ratios, a_1, a_2 are the semi-major axes and e_1, e_2 are the eccentricity of the inner and outer planets respectively. Now we suppose that the stability parameter is $\Gamma_s = r\text{ap} - Y$, and let $\mu_m = \max(\mu_1, \mu_2)$, where

$$\Gamma_s = \frac{a_2(1-e_2)}{a_1(1+e_1)} - 2.4\mu_m^{\frac{1}{3}}\left(\frac{a_2}{a_1}\right)^{\frac{1}{2}} - 1.15 \qquad (20)$$

The planetary system is stable if $\Gamma_s > 0$ otherwise the system is expected to be unstable. According to Petrovich the fate of the unstable systems will depend on the planetary masses i.e., when $\mu_1 > \mu_2$, system lead to ejections and for $\mu_1 < \mu_2$, there is a slightly favouring of collisions with the star. Now for HD 45364 system, we obtain $\mu_1 = 0.000217807$, $\mu_2 = 0.000765467$ and $\Gamma_s = -0.384626 < 0$ and hence we can conclude that this system is unstable and there is a slightly favouring of collisions with the star than ejections.

7 Conclusions

In this paper, we have used three-body problem to analyze the dynamics of HD 45364 exoplanetary system. We have obtained the solutions of perturbation equations for orbital elements. We have plotted the two resonant angles associated with 3:2 MMR and it is found that both the angles are librating around a constant value. This indicates that there exists nearly 3:2 MMR between HD 45364b and HD 45364c. We have studied the effect of planetary perturbations on semi-major axis. The perturbative solutions for the time variation of the semimajor axis are shown for the short and long time for both inner and outer planets of HD 45364 system. For the validation of our analytical results we have compared our analytical solutions with the solutions obtained by numerical methods. We have also studied the effect of planetary perturbations on eccentricity by secular resonance dynamics. We have observed that the periodicity occurs in the variation of eccentricities for both the planets and it is found that eccentricities for both the planets reach its maximum value at the same time and conversely when one eccentricity is minimum, then other reaches exactly its minimum value. Moreover, the eccentricity of HD 45364b oscillates between 0.07 and 0.2147 and eccentricity of HD 45364c oscillates between 0.0976 and 0.1363. We have also observed the long-term variation of the eccentricities over a time span of 10000 yrs and we found similar types of behaviours in the variation of eccentricities of HD 45364b and HD 45364c for long time also. Moreover, using the latest empirical stability criteria given by Petrovich (2015), the dynamical stability of HD 45364 system is studied and it is found that this system is unstable and there is a slightly favouring of collisions with the star other that ejections.

Declarations

Ethical Approval: Not applicable
Competing interests : The authors have no competing interests to declare that are relevant to the content of this article.
Authors' contributions : **Rajib Mia** : Conception and design of study, Methodology, Drafting the manuscript, Revising the manuscript critically for important intellectual content, Analysis and interpretation of data. **Arjun Kumar Paul**:

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