# Kernel Spatial Fuzzy C-Means Clustering with Swarm Intelligence for Brain MRI Image Segmentation

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Abstract Image segmentation of medical images plays a key role in the development of computeraided diagnosis (CAD) systems. In this paper, we present a medical image segmentation algorithm based on fuzzy c-means clustering algorithm (FCM). The basic FCM algorithm is quite sensitive to noise and to alleviate this problem the algorithm is modified by incorporating spatial information and a kernel-induced distance. This new variant of FCM is called kernel spatial FCM (KSFCM). The segmentation result largely depends on the cluster centres, therefore, optimization of the cluster centre is also the main issue that we dealt with in this work by employing swarm intelligence-based algorithms to obtain the cluster centres. We have considered the two most popular swarm-based algorithms i.e. particle swarm optimization (PSO) and artificial bee colony (ABC) and the newly proposed algorithm is called PSOKSFCM and ABCKSFCM respectively. The algorithms are testified by segmentation of brain magnetic resonance imaging (MRI) data and validated using cluster validity functions. The results proved that the proposed algorithm performs better and improves the segmentation result satisfactorily.

Keywords — Brain magnetic resonance imaging; FCM; image segmentation; KSFCM; Particle swarm optimization; artificial bee colony (ABC).

## I. INTRODUCTION

A pattern recognition system [1] in an image is generally involved with finding a distinct pattern in the image so that the region of interest (ROI) can be separated for a detailed study. It is an important aspect of computer vision that has numerous applications in diverse fields. An image segmentation process can be considered as one of the stages of a pattern recognition system. Image segmentation [2] can be defined as aggregating pixels in regions or segments such that every segment is homogeneous but the union of two segments is non-homogeneous. Image segmentation is subjective and therefore it is difficult to judge an image segmentation algorithm as best. Although decades of research have been devoted to image segmentation problems research in this direction is still needed to develop more robust algorithms suited for general applications. The computer vision algorithms for medical imaging applications has recently generated lots of interest especially for computer-aided diagnosis system (CAD). Image segmentation plays a big role in the successful development of the CAD system and thus the focus on segmentation algorithms for medical images is required.

There are many image segmentation techniques [3] such as thresholding, edge detection region growing, clustering, etc. The clustering-based segmentation techniques are quite popular, especially for medical image segmentation problems. Image clustering belongs to the unsupervised classification method in which the image is partition into clusters. The pixels in the image assigned to the cluster that minimizes the distance between the pixel and the cluster centre. The centroid model-based K- means clustering [4] is a widely used method in many classification problems. The next clustering technique that became very popular is fuzzy C- means (FCM) clustering proposed in [5]. The FCM clustering may be considered as a soft clustering extension of Kmeans in which each data belongs to all the clusters with a certain degree of fuzziness.

The rest of the paper is organized as follows: Sect. 2 discussed some of the related works in FCM based image segmentation techniques particularly for brain MRI segmentation, an overview of FCM clustering technique is discussed in Sect. 3 that includes SFCM and KFCM, A brief discussion on swarm intelligence-based PSO and ABC optimization algorithms are presented, Sect.4, the proposed algorithm i.e. the combination of kernel spatial FCM with swarm intelligence are discussed in Sect. 5, the proposed approach is evaluated on brain MRI images, and comparison with other approaches is discussed in Sect. 6 and finally, the conclusion of the proposed work is given in Sect. 7

## II. RELATED WORKS

The FCM clustering-based solutions for medical image segmentation problems can be traced back to the pioneering work by Bezdek et el. [6]. Thereafter many modifications of the standard FCM algorithm were proposed by many researchers and evaluated for MRI image segmentation. A review of different variants of FCM for MRI segmentation is studied in [7]. A detailed report of the comparative analysis of different methods of brain MRI image segmentation is presented by the authors in [8]. There are several modifications of FCM such as the ones that incorporate spatial or neighbourhood information. The authors in [9] modified FCM to compensate for intensity inhomogeneities and allow the labeling of a pixel to be influenced by its immediate neighbourhood labels. The spatial information is incorporated in the spatial FCM (SFCM) proposed by [10], where the membership function is modified by considering a neighbourhood window. The algorithm showed superiority over traditional FCM in the segmentation of noisy MRI images. The work proposed in [11] takes the output of SFCM segmentation as an initial level set function to produced optimized results through level set evolution. A kernelized FCM (KFCM) is proposed in [12], [13] for MRI image segmentation in which a kernel-induced distance metric is utilized instead of Euclidean distance. The algorithm is more robust to noise compared to FCM.

Nature-inspired optimization algorithms are very popular and they tend to provide better solutions for complex problems as compared to the traditional method. The FCM algorithm can further be improved by incorporating algorithms like genetic algorithm (GA), particle swarm optimization (PSO) and artificial bee colony (ABC) algorithms. Some of the research in this direction can be found in the works presented in [14]-[16]. The combination of SFCM with ABC algorithm for brain MRI segmentation is proposed in [17]. The ABC algorithm tried to obtain an optimized cluster centre with the optimization function derived from SFCM. The key to the success of FCM clustering-based image segmentation methods is obtaining the best clusters. This idea is also utilized in the genetic algorithm KFCM (GAKFCM) clustering algorithm [18] which provides a better clustering result.

It can be observed from the above algorithms that utilization of optimization algorithms can improve the overall segmentation result. In this paper, we proposed to combine swarm intelligence-based algorithms with kernelized spatial FCM (KSFCM) to obtain a better segmentation result in brain MRI images.Size 10 & Normal)An easy way to comply with the conference paper formatting requirements is to use this document as a template and simply type your text into it.

# III. FUZZY BASED CLUSTERING ALGORITHMS

In this section, we highlight the clustering techniques that we use in this work. The algorithms are variants of the basic FCM algorithm.

# A. Fuzzy c means Clustering (FCM)

The FCM algorithm is an optimization algorithm that iteratively aims to produce optimized clusters in an image. The fuzzy membership values are used to determine the cluster of the pixels.

Let us consider an image arranged in a onedimensional matrix as  $X=[x_1, x_2,..., x_N]$ , where  $x_i$ represents pixel intensity value or feature value and N is the total number of pixels in the image. FCM aims to partition the pixels into *c* clusters.

The basic FCM takes the Euclidean distance between and the cluster centre along with the membership values as the cost function, which is to be minimized, as defined in Eq. (1).

$$J = \sum_{j=1}^{N} \sum_{i=1}^{C} u_{ij}^{m} \left\| x_{j} - v_{i} \right\|^{2}$$
(1)

Where  $u_{ij}$  is the degree of membership of pixel  $x_j$  to belong in  $i^{\text{th}}$  cluster,  $v_i$  is the cluster centre,  $\|\cdot\|$  is the Euclidean norm and m is a constant that influences the fuzzy partition. As FCM tries to minimize the objective function the membership values and the cluster centres are updated according to Eq. (2) and Eq. (3) respectively.

$$u_{ij} = \frac{1}{\sum_{k=1}^{C} \left( \frac{\|x_j - v_i\|}{\|x_j - v_k\|} \right)^{\frac{2}{(m-1)}}}$$
(2)  
$$v_i = \frac{\sum_{j=1}^{N} u_{ij}^m x_j}{\sum_{j=1}^{N} u_{ij}^m}$$
(3)

#### B. Spatial FCM (SFCM)

The performance of FCM deteriorates when the image is noisy. The noise in an image may arise from capturing devices or artifacts. To alleviate this problem a spatial FCM is proposed in [10] in which the information of the neighbourhood pixels are exploited. A pixel under consideration is probably correlated with the neighboring pixels. Thus the neighbourhood pixels play a critical role in differentiating a pixel in a neighbourhood. The influence of the neighbourhood pixels on a pixel under consideration is incorporated in the membership value assignment of the pixel in a cluster. The spatial function used in the new membership function is defined as in Eq. (4).

$$h_{ij} = \sum_{k \in NB(x_i)} u_{ik} \tag{4}$$

Where  $NB(x_j)$  is a square window define on the image centered on the pixel  $x_j$ . The size of the window is arbitrary. The spatial function  $h_{ij}$  is defined as the probability that a pixel  $x_j$  belongs to the i<sup>th</sup> cluster. If the majority of the neighbourhood pixels belong to a cluster then it indicates a higher value of the centre pixel in that cluster. Accordingly, the modified membership is obtained in Eq. (5).

$$u_{ij}^{'} = \frac{u_{ij}^{p} h_{ij}^{q}}{\sum_{k=1}^{C} u_{kj}^{p} h_{kj}^{q}}$$
(5)

Where p and q are parameters that control the contribution of the original membership values and the spatial function. The spatial FCM is denoted by SFCM<sub>*p*,*q*</sub>. When *p*=1 and *q*=0 SFCM degenerates to conventional FCM.

# C. Kernelized FCM (KFCM)

The concept of the kernel is introduced in FCM to modify the Euclidean distance function in the objective function of basic FCM. The distance between a data and the cluster centre is calculated in the kernel space by way of mapping using a kernel function. Accordingly, the objective function of kernelized FCM is defined as in Eq. (6).

$$J = \sum_{k=1}^{C} \sum_{i=1}^{N} \mu_{ki}^{m} \| \varphi(\mathbf{x}_{k}) - \varphi(\mathbf{v}_{i}) \|^{2}$$
(6)

$$\left\| \varphi(\mathbf{x}_k) - \varphi(\mathbf{v}_i) \right\|^2 = K(\mathbf{x}_k, \mathbf{x}_k) + K(\mathbf{v}_i, \mathbf{v}_i) - 2K(\mathbf{x}_k - \mathbf{v}_i)$$

$$(7)$$

Where  $K(x,v) = \varphi(x)^T \varphi(x)$  is an inner product kernel function. There are several kernel functions but we consider Gaussian kernel as defined in Eq. (8).

$$\boldsymbol{K}(\boldsymbol{x},\boldsymbol{\nu}) = \exp\left(\frac{-\left|\boldsymbol{x}-\boldsymbol{\nu}\right|^2}{\sigma^2}\right)$$
(8)

Then by substituting it in Eq. (7) and subsequently in Eq. (6), the objective function is modified as defined below.

$$J = \sum_{k=1}^{C} \sum_{i=1}^{N} \mu_{ki}^{m} (1 - K(x_{k}, v_{i}))$$
(9)

The objective function defined in Eq. (9) is to be minimized with respect to the membership function and accordingly the updated membership function and the new cluster centres are obtained in Eq. (10) and Eq. (11) respectively.

$$\mu_{ki} = \frac{\left(\frac{1}{1 - K(x_k, v_i)}\right)^{\frac{1}{m-1}}}{\sum_{j=1}^{C} \left(\frac{1}{1 - K(x_k, v_j)}\right)^{\frac{1}{m-1}}}$$
(10)

$$v_{i} = \frac{\sum_{k=1}^{C} \mu_{ki}^{m} K(x_{k}, v_{i}) x_{k}}{\sum_{k=1}^{N} \mu_{ki}^{m} K(x_{k}, v_{i})}$$
(11)

## IV. SWARM INTELLIGENCE BASED OPTIMIZATION METHODS

Nature-inspired population-based optimization algorithms are quite popular in solving many engineering and non-engineering problems. So in this work, we have incorporated swarm intelligencebased methods to find the optimized cluster centres in the image segmentation problem. We have considered two of the most popular and efficient algorithms i.e. particle swarm optimization (PSO) and artificial bee colony (ABC) methods. We present a summary of these algorithms.

#### A. Particle Swarm Optimization (PSO)

Particle swarm optimization is an optimization algorithm introduced by Kennedy and Eberhart [19]. It is a simple and very effective optimization algorithm in which a population of agents called particles is employed and the position of the particles represents the solution of a problem. The movement of the particles is guided by a fitness function and the velocity and position of the particles for the next iteration are calculated using Eq. (12) and Eq. (13) respectively.

$$v_i(t+1) = w.v_i(t) + c_1.(p_i(t) - x_i(t)) + c_2.(p_o(t) - x_i(t))$$
(12)

$$x_i(t+1) = x_i(t) + v_i(t)$$
 (13)

where  $v_i(t)$  and  $x_i(t)$  represents velocity and position of  $i^{\text{th}}$  particle;  $p_i(t)$  and  $p_g(t)$  are local best and global best obtained in one step; w is constant called inertia which decides the importance of the previous velocity;  $c_1$  and  $c_2$  are acceleration constants which acts as a weight for the local best and global best.

#### B. Artificial Bee Colony (ABC)

A unique food source searching methodology followed by real bees inspired Artificial Bee Colony Algorithm (ABC) optimization algorithm [20]. The bees are divided into three groups based on the assigned task: employed bee, onlooker bee, and scout bee. The bees search for food and the quality of the food source is ascertained from the amount of nectar. In the ABC algorithm, a candidate for a solution is represented by a food source position. The strength of the solution is determined by the fitness function of the problem. The equations that governed the update of the solution in the next iteration are given as follows.

$$x_{i}(t+1) = x_{i}(t) + \phi_{i} \cdot (x_{i}(t) - x_{m}(t))$$
(14)

Where  $x_i(t)$  is the *i*<sup>th</sup> food source position or solution at the time 't', 'm' is randomly chosen index different from 'i'. It determines the neighbour for updating the *i*<sup>th</sup> solution.  $\phi_i$  is a random number in the interval [-1,+1] which acts as a weighting factor. A probability value is used for selection of *i*<sup>th</sup> selection for the employed bees and the onlooker bees.

$$p_i = \frac{f(x)}{\sum_{m=1}^{S} f(x_m)}$$
(15)

Where  $f(x_i)$  is the fitness value of the solution  $x_i$ .

# V. COMBINED KERNELIZED FCM (KFCM) WITH SWARM INTELLIGENCE

In this section, we present the proposed algorithm which is a combination of kernelized FCM with spatial constraints and swarm intelligence optimization algorithms. The conventional FCM is very sensitive to noise or to an outlier so to deal with this problem two methods are employed to upgrade FCM. One method is to replace the Euclidean distance with a kernel-induced distance metric which performs better to outlier pixels and the other approach is to take into consideration the influence of neighbourhood pixels i.e. spatial information. This combined algorithm is called kernelized spatial FCM (KSFCM).

The conventional FCM is similar to a local hillclimbing algorithm that makes it sensitive to the initial clustering centre and the cluster centre may get trapped to a local extremum. A global search algorithm if incorporated with FCM may lead to a global best value and this requirement can be fulfilled by swarm-based search algorithms. We utilize the ability of PSO and ABC algorithms to obtain the optimized clusters and in combination with KSFCM two new algorithms are proposed which are called PSOKSFCM and ABCKSFCM. Either PSO or ABC will obtain the optimized cluster centre by minimizing an objective function which is designed by modifying Eq. (6) as follows:

$$J_{KSFCM} = \sum_{t=1}^{C} \sum_{i=1}^{N} \mu_{ksi}^{m} \left\| \varphi(x_{t}) - \varphi(v_{i}) \right\|^{2}$$
(16)

Where  $\mu_{ksi}$  is the membership function that includes spatial information and it can be obtained from Eq. (5). The description of the proposed algorithm is as shown in Fig. 1.

begin         {         initialize number of clusters         initialization of membership function         Set Parameters of PSO or ABC         swarm intelligence ()         {         initialize random cluster centre for all the populations         repeat         {         calculate the objective function J <sub>KSFCM</sub> for all the         populations
calculate the kernalized membership $\mu_{ki}$
calculate spatial membership $\mu_{ksi}$ update the cluster centres $v(t+1)$ using PSO or ABC if (convergence reached) exit (); else repeat
}

# Fig. 1: Proposed PSOKSFCM/ABCKSFCM Algorithm

# VI. RESULTS AND DISCUSSION

The proposed scheme is illustrated with examples of brain MRI image segmentation. To show the effectiveness of the proposed scheme (PSOKSFCM and ABCKSFCM) we carried out a comparative analysis with KFCM and KSFCM. Cluster validity functions are used to indicate the success level of clustering. These functions help in comparative performance analysis. Therefore, in this study, we have used three validity indices namely, fuzzy partition coefficient,  $V_{pc}[21]$  fuzzy partition entropy,  $V_{pe}[22]$ , and Xie-Beni validity function,  $V_{xb}[23]$  which are defined as follows:

$$V_{pc} = \frac{\sum_{j=1}^{N} \sum_{i=1}^{C} u_{ij}^{2}}{N}$$
(17)

$$V_{pe} = \frac{-\sum_{j=1}^{N} \sum_{i=1}^{C} \left( u_{ij} \log u_{ij} \right)}{N}$$
(18)

$$V_{xb} = \frac{-\sum_{j=1}^{N} \sum_{i=1}^{C} u_{ij} \|x_j - v_i\|^2}{N * (\min_{i \neq k} \{\|v_k - v_i\|^2\})}$$
(19)

These indices give a quantitative assessment of how good a clustering result is. When  $V_{pc}$  is maximal or

 $V_{pe}$  is minimal the clustering algorithm is the best. The correlation between a fuzzy partition and the feature attribute of the data is exploited in the validity function  $V_{xb}$ . It is considered to be a better validity function compared to the other two. When  $V_{xb}$  is minimum good clustering result is produced.

The experiments are carried out in three different examples. In the first experiment, a normal brain MRI [11] is segmented into three clusters to highlight the three brain regions i.e. white matter (WM), grey matter (GM), and cerebrospinal fluid (CSF). The segmentation results are shown in Fig. 2. The number cluster for this experiment is set to 3 since we are interested in the segmentation of three regions of the normal brain from the MRI image. It can be observed in Fig. 2 that PSOKSFCM and ABCKSFCM algorithms can perform better and the detailed information including the edges are preserved in the segmentation result. The results are further validated by the validity functions values given in Table 1. In the second experiment, we have taken two examples from clinical brain MR images with tumours. Two T-1 weighted axial slices with 240 x 240 pixels were taken from MICCAI BRATS2014 challenge 2014 [24]. We set the number of the cluster for Brats1 and Brats 2 images to 3 and 4 respectively. The observation that can be made from the segmentation results shown in figure3 and figure 4 is that the tumour region from the image can be well separated with betterpreserved boundaries. The quantitative measures of the segmentation results given by the validity functions support the superiority of swarm intelligence-based kernelized spatial FCM.



Fig. 2: Segmentation results on normal brain MRI [11]: (a) Original image (b) KFCM (c) KSFCM (d) PSOKSFCM (e) ABCKSFCM



Fig. 3: Segmentation results on Brats1 image: (a) Brats1 (b) KFCM (c) KSFCM (d) PSOKSFCM (e) ABCKSFCM



Fig. 4: Segmentation results on Brats2 image: (a) Brats2 (b) KFCM (c) KSFCM (d) PSOKSFCM (e) ABCKSFCM

TABLE I Validity indices for different algorithms

Images	Methods	Vpc	Vpe	Vxb
Normal	SFCM	0.9224	0.0566	0.0722
brain	KFCM	0.869	0.101	0.0686
MRI	KSFCM	0.9239	0.0554	0.0719
	PSOKSFCM	0.9195	0.0579	0.0630
	ABCKSFCM	0.9198	0.0579	0.0667
Brats1	KFCM	0.909	0.070	0.0914
	KSFCM	0.9236	0.0577	0.0911
	PSOKSFCM	0.9438	0.0441	0.0471
	ABCKSFCM	0.9439	0.0441	0.0480
Brats2	KFCM	0.916	0.072	0.0395
	KSFCM	0.9567	0.0366	0.0193
	PSOKSFCM	0.9562	0.0369	0.0181
	ABCKSFCM	0.9566	0.0369	0.0184

## VII. CONCLUSION

In this paper, we have proposed a combination of swarm intelligence with kernelized FCM for brain MRI image segmentation problems The basic FCM algorithm has been modified KSFCM to include the spatial information and by changing the Euclidean distance metric to kernel induced distance. The swarm intelligence is incorporated to generate optimized cluster centres. The proposed algorithms performed better when tested on a normal brain MRI image and two other brain MRI images with a tumour. The success of the algorithms is validated by the cluster validity functions. The only drawback of using swarm intelligence is perhaps the computational cost but achieving good segmentation results especially for medical images is more desirous. The proposed algorithm is a good solution for MRI segmentation problem and the result can be used for further processing in the overall design of a computer-aided diagnosis system.

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